General Instructions:
(i) All questions are compulsory.
(ii) Figures to the right indicate full marks.
*(iii) The question paper consists of **11** questions into **FOUR** sections **A,B,C,D.***
- ***Section A** contains **2** questions of **1 mark** each.*
- ***Section B** contains **3** questions of **2 marks** each.*
- ***Section C** contains **2** questions of **3 marks** each.*
- ***Section D** contains **4** questions of **4 marks** each.*

SECTION A

Select and write the most appropriate answer from the given alternatives for each question:

1. Which of the following is a statement? **[1]**
(A) Do you like Mathematics? (B) Hurry up!
(C) Will you help me? (D) The earth rotates around sun

2. If $p \wedge q = F$, $p \to q = F$, then the truth value of p and q is **[1]**
(A) T, T (B) T, F
(C) F, T (D) F, F

SECTION B

3. Write the converse and contrapositive of the statement: "If two triangles are congruent, then their areas are equal. " **[2]**

4. Find the converse and inverse of the statement "The surface area decreases then pressure increases." **[2]**

5. Examine whether the following logical statement pattern $\left[(p \to q) \wedge q\right] \to p$ is tautology, contradiction or contingency. **[2]**

SECTION C

6. Prepare the truth table for the following: **[3]**
(a) $\sim p \wedge q$
(b) $p \to (p \vee q)$
(c) $\sim p \leftrightarrow q$

7. Without using the truth table, show that $\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$ **[3]**

SECTION D

8. Using truth table, show that: $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$ **[4]**

9. Determine whether the following statement pattern is a "tautology" or "contradiction" or "neither" of the two: **[4]**
$(\sim p \vee q) \rightarrow p \wedge (q \vee \sim q)$

10. Without using truth table show the following: **[4]**
(a) $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$
(b) $(p \wedge q) \vee (\sim p \wedge q) \vee (\sim q \wedge r) \equiv q \vee r$

11. Write the negations of the following stating the rules used. **[4]**
(a) $(p \vee q) \wedge (q \vee \sim r)$
(b) $(p \rightarrow q) \vee r$
(c) $p \wedge (q \vee r)$
(d) $(\sim p \wedge q) \vee (p \wedge \sim q)$

Answers

1 (D)

Statement: The earth rotates around sun

2 (D)

$p \wedge q = F,$

$p \to q = F \equiv \sim p \vee q = F$

$(D)\, F, F$

3

Converse: $q \to p$

If areas of two triangles are equal, then they are congruent.

Contrapositive: $\sim q \to \sim p$

If areas of two triangles are not equal, then they are not congruent.

4

Converse: $q \to p$

If the pressure increases then the surface area decreases.

Inverse: $\sim p \to \sim q$

The surface area does not decreases then the pressure does not increase.

5

Given logical statement is

$\left[(p \to q) \wedge q\right] \to p$

Truth table for above logical statement

p	q	$p \to q$	$(p \to q) \wedge q$	$\left[(p \to q) \wedge q\right] \to p$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	F
F	F	F	F	F

By the observation of above truth table, it is clear that given logical statement is a contingency.

6

(a) $\sim p \wedge q$

p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

(b) $p \to (p \vee q)$

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

(c) $\sim p \leftrightarrow q$

p	q	$\sim p$	$\sim p \leftrightarrow q$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	F

7

$\sim (p \vee q) \vee (\sim p \wedge q)$

$\equiv \sim (p \vee q) \vee \sim (p \vee \sim q)$ [By De-Morgan's Law]

$\equiv \sim \left[(p \vee q) \wedge (p \vee \sim q)\right]$ By De-Morgan's Law]

$\equiv \sim \left[\{(p \vee q) \wedge p\} \vee \{(p \vee q) \wedge \sim q\}\right]$

[By Distributive Law]

$\equiv \sim \left[\{p\} \vee \{(p \vee q) \wedge \sim q\}\right]$ [By Absorption Law]

$\equiv \sim \left[\{p\} \vee \{(p \wedge \sim q) \vee (q \wedge \sim q)\}\right]$

[By Distributive Law]

$\equiv \sim \left[\{p\} \vee \{(p \wedge \sim q) \vee \mathrm{F}\}\right]$ [By Complement Law]

$\equiv \sim \left[\{p\} \vee (p \wedge \sim q)\right]$ [By Identity Law]

$\equiv \left[\{\sim p\} \wedge (\sim p \vee q)\right]$ [By De-Morgan's Law]

$\equiv \sim p$ [By De-Morgan's Law]

8

The truth table for the given logical statement is:

1	2	3	4	5	6	7	8	9
p	q	$\sim p$	$\sim q$	$p \leftrightarrow q$	$\sim (p \leftrightarrow q)$	$p \wedge \sim q$	$q \wedge \sim p$	$(p \wedge \sim q) \vee (q \wedge \sim p)$
T	T	F	F	T	F	F	F	F
T	F	F	T	F	T	T	F	T
F	T	T	F	F	T	F	T	T
F	F	T	T	T	F	F	F	F

The entries in the columns 6 and 9 are identical.

9

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$q \vee \sim q$	$p \wedge (q \vee \sim q)$	$(\sim p \vee q) \rightarrow p \wedge (q \vee \sim q)$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	T	F	F
F	F	T	T	T	T	F	F

In the above truth table, all the entries in the last column are combination of T and F.

Thus, the logical statement is a contingency.

10

(a)

L.H.S. =

$p \leftrightarrow q$

$\equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$\equiv (p \vee \sim q) \wedge (q \vee \sim p)$

$\equiv ((p \vee \sim q) \wedge q) \vee ((p \vee \sim q) \wedge \sim p)$

$\equiv ((p \wedge q) \vee (\sim q \wedge q)) \vee ((p \wedge \sim p) \vee (\sim q \wedge \sim p))$

$\equiv ((p \wedge q) \vee (F)) \vee ((F) \vee (\sim q \wedge \sim p))$

$\equiv (p \wedge q) \vee (\sim p \wedge \sim q) =$ R.H.S.

(b)

$(p \wedge q) \vee (\sim p \wedge q) \vee (\sim q \wedge r)$

$\equiv [(p \vee \sim p) \wedge q] \vee (\sim q \wedge r)$ [By Associative and Distributive law]

$\equiv [\mathrm{T} \wedge q] \vee (\sim q \wedge r)$ [By Complement law]

$\equiv q \vee (\sim q \wedge r)$ [By Identity law]

$\equiv (q \vee \sim q) \wedge (q \vee r)$ [By Distributive law]

$\equiv (\mathrm{T}) \wedge (q \vee r)$ [By Complement law]

$\equiv q \vee r$ [By Identity law]

11

(a)

$\sim [(p \vee q) \wedge (q \vee \sim r)]$

$\equiv \sim (p \vee q) \vee \sim (q \vee \sim r)$ [By DeMorgan's law]

$\equiv (\sim p \wedge \sim q) \vee (\sim q \wedge r)$ [By DeMorgan's law]

$\equiv (\sim q \wedge \sim p) \vee (\sim q \wedge r)$ [By Commutative law]

$\equiv \sim q \wedge (\sim p \vee r)$ [By Distributive law]

(b)

$\sim [(p \rightarrow q) \vee r]$

$\equiv \sim (p \rightarrow q) \wedge \sim r$ [By DeMorgan's law]

$\equiv \sim (\sim p \vee q) \wedge \sim r$

$\equiv (p \wedge \sim q) \wedge \sim r$ [By DeMorgan's law]

(c)

$\sim [p \wedge (q \vee r)]$

$\equiv \sim p \vee \sim (q \vee r)$ [By DeMorgan's law]

$\equiv \sim p \vee (\sim q \wedge \sim r)$ [By DeMorgan's law]

(d)

$\sim [(\sim p \wedge q) \vee (p \wedge \sim q)]$

$\equiv \sim (\sim p \wedge q) \wedge \sim (p \wedge \sim q)$ [By DeMorgan's Law]

$\equiv (p \vee \sim q) \wedge (\sim p \vee q)$ [By DeMorgan's Law]